

B.A/B.Sc - 6th Semester

(2721)

Paper: Mathematics Paper-II (Numerical Analysis)

Time Allowed: 2 hrs.

Max. Marks: 50

Note: There are EIGHT questions of equal marks. Candidates are required to attempt any FOUR questions. Non-programmable scientific calculator is allowed.

SECTION—A

1(a) Using generalized Newton's method find a double root of the equation $f(x) = x^3 - x^2 - x + 1 = 0$.

(b) Establish an iteration formula to find the reciprocal of a positive number N by Newton-Raphson method. Hence find the reciprocal of 154 to four significant figures.

2(a) Decompose the matrix

$$A = \begin{pmatrix} 4 & 3 & 2 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{pmatrix}$$

into the form LU , where L is the lower triangular and U is unit upper triangular matrix.

(b) Solve the system

$$10x_1 + 2x_2 + x_3 = 9; \quad 2x_1 + 20x_2 - 2x_3 = -44; \quad -2x_1 + 3x_2 + 10x_3 = 22$$

by Gauss-Seidel method.

Section—B

3(a) Define the operators Δ , ∇ , δ , E and E^{-1} and show that

(i) $\Delta^2 = (1 + \Delta)\delta^2$

(ii) $\nabla = \delta E^{-1/2}$

(b) Locate and correct the error in the following table

4(a) Form a table of differences for the function $f(x) = x^3 + 5x - 7$ for $x = -1, 0, 1, 2, 3, 4, 5$.

Consider the table to obtain $f(6)$ and $f(7)$.

(b) Prove that $\Delta^r y_k = \nabla^r y_{k+r} = \delta^r y_{k+r/2}$.

Contd....P/2

(2)

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	2.7183	2.8577	3.0042	3.1528	3.3201	3.4903	3.6693

Section—C

5(a) State Stirling's formula for interpolation at middle of a table of values and find $e^{1.91}$ from the following table.

x	1.7	1.8	1.9	2.0	2.1	2.2
e^x	5.4739	6.0496	6.6859	7.3891	8.1662	9.0250

(b) Write an algorithm for Lagrange's formula. Find the polynomial which fits the following data

$(-1, 7)$, $(1, 5)$ and $(2, 15)$.

6(a) From the following values of x and y ,

x	0	1	2	3	4	5	6
y	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

find (i) $\frac{dy}{dx}$, when $x = 1$

(ii) $\frac{d^2y}{dx^2}$ at $x = 3$.

(b) Evaluate $\int_0^\pi x \sin x \, dx$ using trapezoidal rule with five ordinates.

SECTION—D

7(a) Given the differential equation $\frac{dy}{dx} = x^2 + y$ with $y(0) = 1$, compute $y(0.02)$ with Euler's modified method.

(b) Evaluate $\int_0^1 \int_0^1 e^{x+y} \, dx \, dy$ with trapezoidal and Simpson's rule with $h = k = 0.5$.

8(a) Use Picard's method to obtain $y(0.1)$ and $y(0.2)$ of the problem defined by

$$\frac{dy}{dx} = x + yx^4, \quad y(0) = 3.$$

(b) If $\frac{dy}{dx} = \frac{1}{x^2 + y}$, with $y(4) = 4$, compute the values of $y(4.1)$ and $y(4.2)$ with Taylor's series method.